New Results in Generalized Minimum Variance Control of Computer Networks

Wojciech Przemyslaw Hunek

Institute of Control and Computer Engineering, Department of Electrical, Control and Computer Engineering
Opole University of Technology
ul. Sosnkowskiego 31, 43-272 Opole, Poland
e-mail: w.hunek@po.opole.pl

Piotr Dzierwa

Institute of Control and Computer Engineering, Department of Electrical, Control and Computer Engineering
Opole University of Technology
ul. Sosnkowskiego 31, 43-272 Opole, Poland
e-mail: piotr.dzierwa@yahoo.com

crossref http://dx.doi.org/10.5755/j01.itc.43.3.6268

Abstract. In this paper new results in adaptive (generalized) minimum variance control of packet switching computer networks are presented. New solutions, corresponding to the new inverses of the nonsquare polynomial matrices, can be used for design of robust control of multivariable systems with different number of inputs and outputs. Application of polynomial matrix inverses with arbitrary degrees of freedom creates the possibilities to optimal control of computer networks in terms of usage their maximal bandwidth. Simulation examples made in Matlab environment show big potential of presented approach.

Keywords: generalized minimum variance adaptive control; packet switching networks; active queue management; packet queue management; robust control; Drop/Mark Probability Function.

1. Introduction

Dynamic development of high-speed data transmission determined the process of improving the reliability of computer networks. This problem is mainly dealt with by intelligent packet switching systems. Meeting high quality standards regulated by the Quality of Service (QoS) becomes more and more difficult these days and the problem of ensuring the efficient operation of the network is one of the most important problems of the modern Internet.

A wide variety of mechanisms and solutions implementing efficient operation of packet switching networks is used for the stable operation of the packet networks. The solutions based on queuing mechanisms [1] may be included among the most popular ones. The classic way of queue management uses First-In/First-Out (FIFO) mechanism to organize the work of queue buffer. More advanced methods of queue management often use Active Queue Management (AQM) [2-5], that shapes the traffic using different techniques, such as drop-tail mechanism [6]. Equally efficient, interesting AQM solutions involving one-dimensional so-called Single-Input/Single-Output (SISO) computer network models can be found in the literature [7-10].

A slightly different solution to the mentioned problems involves the use of neural networks [4], fuzzy logic [11,12] and neuro-fuzzy approach [13]. However, the more intriguing seems to be the use of analytical methods to better describe the reality observed and to enable more efficient estimation of the tested object/process parameters. Such an approach can be found in [14], which uses a known SISO Generalized Minimum Variance Control (GMVC) algorithm [15]. On the other hand, the solution presented in [10] is based on Minimum Variance Adaptive Control (MVAC) algorithm.

There are many tools to simulate the dynamics of computer networks in different operating conditions. These classic, dedicated environments may include the NS-2 [16] or object-oriented OMNeT++ [17-20].
2. Problems of packet switching

The rapid growth of computer networks has increased the requirements on the operation speed and accuracy of the active devices found in the computer network structure. The key issue seems to be deadlock problems [10,14]. Bottlenecks effectively inhibit the free data flow with the maximum available bandwidth. The efficient and fast packet scheduling algorithm, which can guarantee the quality of streaming, becomes the sensitive spot of a network. It seems that one of the few such algorithms is queuing mechanism, designed to meet the restrictions of efficient computer networks operation.

The FIFO mechanism, mentioned in the Introduction, is a typical method of queue management in network routers, where the network device caches the maximum number of incoming packets cutting off those redundant packets. Managing queue using this algorithm allows for the possibility of a deadlock in case of frequent network buffer overflow. However, more advanced queue management methods can be found in literature [20,21]. The Random Early Detection (RED) algorithm also seems to be an alternative to the FIFO.

RED queuing algorithm reduces the problem of network deadlock by monitoring average buffer queue. It is based on the determination of the statistical probabilities of packet rejection. The probability close to zero means that the buffer is empty and it is permitted to receive packets from subnet. During the process of filling the buffer, the probability of packet rejection increases, with the numerical value of 1 when the buffer is full. The use of RED algorithms guarantees earlier deadlock detection, providing QoS for users at the same time.

There are various methods to resolve the problem of efficient management of packet switching and thus to improve network performance. An approach based on Cellular Neural Network (CNN) [11] can be effectively applied to the real-time scheduling of the cells in high-speed ATM commutators, particularly in the commutators based on the matrix architecture with the output buffering virtualization. The quoted method is, however, dedicated to ATM networks and its applicability is reduced to restrictions resulting from the characteristics of the cellular neural network itself.

On the other hand, the classical neural network [21] can also be used as a relevant tool for solving problems related to the efficient operation of computer networks. The ability to adapt to changing operating conditions of different network architectures favors the use of this interesting approach. Unfortunately, the reality mapping models are here created by way of adaptation, often based on incomplete expert knowledge. Therefore, the neural network may not meet the restrictive packet switching time frames. Another approach, no less interesting solution, is based on the application of uncertain variables to assess the stability of congestion control systems for ATM networks [12]. This proposal is based on the distribution of uncertainty given by the experts for the object, which is the entire studied computer network. Load control is done here using RM cells and sources transmission monitoring – Available Bit Rate (ABR). The model implementing this approach is a model without delay, which slightly simplifies the reality observed, paving the way for other methods, not necessarily connected with ATM technology.

In summary, a number of mechanisms to improve the performance of the packet switching networks can be found in the literature. Most of them are based on a heuristic approach, a small number uses analytical tools. This paper is an attempt to transfer complex, as found in modern control theory, minimum variance control algorithms and to confront the new approach with the mechanisms outlined above. In particular, there will be Multi-Input/Multi-Output (MIMO) both adaptive minimum variance control and adaptive generalized minimum variance control algorithms. The use of analytical RED algorithms seems to be an alternative and even competition with existing solutions dedicated to deal with issues of performance of optimum computer networks. It is significant that the new approach can be used not only in TCP/IP networks, but also in networks, where there is no data retransmission. This is confirmed by extensive research, partly illustrated by examples of simulation in the present work.

3. Selection of queuing model

Linear Time-Invariant (LTI) ARMAX model has been proposed to describe the dynamics of packet switching in multivariable networks with the \( n_u \)-input and the \( n_y \)-output

\[
A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})e(t),
\]

where polynomial matrices \( B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_nq^{-n} \), \( A(q^{-1}) = a_{n_y}q^{-1} + \cdots + a_0 \), and \( C(q^{-1}) = c_{n_y}q^{-1} + \cdots + c_0q^{-k} \) (in the backward shift operator \( q^{-1} \)) are of the dimensions \( n_y \times n_u \times n_y \) and \( n_y \times n_y \) respectively, and \( e(t) \) is the uncorrelated zero-mean disturbance at (discrete) time \( t \), and \( d = n - m \) is the time delay of the system. Assume that the transfer-function matrix \( G \in \mathbb{R}^{n_y \times n_x}(z) \) in the complex operator \( z \) can be represented in the Matrix Fraction Description (MFD) form

\[
G(z) = A^{-1}(z)B(z) \quad \text{with} \quad A(z) = z^nI_{n_y} + \cdots + a_n \text{ and } B(z) = z^n b_0 + \cdots + b_m, \] Where \( n, m \) are the orders of the respective matrix polynomials and \( I_{n_y} \) is the identity \( n_y \)-matrix. Alternatively, we can write

\[
A(z^{-1}) = I_{n_y} + \cdots + a_{n_y}z^{-n} \quad \text{and} \quad A(z) = z^nA(z^{-1}) \]

also \( B(z^{-1}) = b_0 + \cdots + b_mz^{-m} \) and \( B(z) = z^m \)
New Results in Generalized Minimum Variance Control of Computer Networks

Let \( B(z^{-1}) \) with \( G(z) = A^{-1}(z)B(z) = z^{-d}A^{-1}(z^{-1})B(z^{-1}) \). Assume that our adaptive TCP/AQM process is described by the model

\[
\Delta(q^{-1})y(t) = q^{-d}B(q^{-1})p(t) + \zeta(q^{-1})e(t), \quad (2)
\]

where \( \Delta(q^{-1}) = 1 + \sum_{\ell=1}^{d} \gamma_{\ell} q^{-\ell}, B(q^{-1}) = \beta_{p} \zeta(q^{-1}) = 1 + \sum_{\ell=1}^{d} \gamma_{\ell} q^{-\ell} \), \( d = 1 \) and \( q(t) \) is the queue length, while \( p(t) \) is the Drop/Mark Probability Function [10]. Parameter estimation for \( q(t) \) is carried out in accordance with the Recursive Least Squares (RLS) algorithm modified by Kaczmarz (generally called the normalized projection algorithm [23])

\[
\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{y_p(t)}{\alpha + \phi_t^T(p(t))} \left[ q(t) - \phi_t^T(\hat{\theta}(t-1)) \right], \quad (3)
\]

where \( \hat{\theta}(t) \) is an estimate of the vector of model parameters \( \theta(t) = [a \ b \ c]^T \), \( \phi(t) = \left[ -q(t) \ p(t-1) \ e(t-1) \right]^T \), \( e(t) = q(t) - \phi_t^T(t-1) \theta(t-1) \) and \( y \in (0,2) \) and \( \alpha \geq 0 \) are the step length of the parameters adjustment and the positive constant operates in case of \( \phi_t^T(t) \phi(t) = 0 \). The scheme of packet switching control system in computer networks is presented later in the paper.

4. Minimum variance control algorithms

4.1. Closed-loop discrete-time minimum variance adaptive control (DMVAC)

The DMVAC law, minimizing the performance index

\[
\min_{u(t)} E \left\{ \left[ y(t+d) - y_{ref}(t+d) \right]^T \times \left[ y(t+d) - y_{ref}(t+d) \right] \right\}, \quad (4)
\]

where \( E \{ \cdot \} \) is the expectation operator and \( y(t+d) = \tilde{\zeta}^{-1}(q^{-1}) \bar{F}(q^{-1}) B(q^{-1}) u(t) + \bar{H}(q^{-1}) y(t) + \tilde{F}(q^{-1}) e(t) \) and \( y_{ref}(t+d) \) are the stochastic output predictor and the reference/settingpoint, respectively, is of the form [24,25]

\[
u(t) = \mathbf{B}^R(q^{-1}) y(t), \quad (5)
\]

with

\[
y(t) = \tilde{F}^{-1}(q^{-1}) \left[ \tilde{\zeta}(q^{-1}) y_{ref}(t+d) - \bar{H}(q^{-1}) y(t) \right].
\]

The appropriate polynomial \((n_y \times n_y)\) -matrices \( \tilde{F}(q^{-1}) = I_{n_y} + \tilde{F}_1 q^{-1} + \cdots + \tilde{F}_{d-1} q^{-d+1} \) and \( \bar{H}(q^{-1}) = \bar{H}_0 + \bar{H}_1 q^{-1} + \cdots + \bar{H}_{n_y-1} q^{-n_y+1} \) are computed from the polynomial matrix identity (called the Diophantine Equation)

\[
\tilde{\zeta}(q^{-1}) = \tilde{F}(q^{-1}) \Delta(q^{-1}) + q^{-d} \bar{H}(q^{-1}), \quad (6)
\]

and

\[
\tilde{\zeta}(q^{-1}) \tilde{F}(q^{-1}) = \tilde{F}(q^{-1}) \tilde{\zeta}(q^{-1}), \quad (7)
\]

where

\[
\tilde{F}(q^{-1}) = I_{n_y} + f_1 q^{-1} + \cdots + f_{d-1} q^{-d+1} \quad \text{and} \quad \bar{H}(q^{-1}) = \bar{H}_0 + \bar{H}_1 q^{-1} + \cdots + \bar{H}_{n_y-1} q^{-n_y+1}.
\]

For right-invertible systems, the symbol \( \mathbf{B}^R(q^{-1}) \) denotes, in general, an infinite number of right inverses of the numerator polynomial matrix \( \mathbf{B}(q^{-1}) \) (see e.g. [26-29]).

Now, for our queueing model we have

\[
\min_{p(t)} E \left\{ \left[ y(t+d) - q_{ref}(t+d) \right]^2 \right\}, \quad (8)
\]

where \( y_{ref}(t+d) = \bar{q}_{ref}(t+d) \) is arbitrary chosen reference queue length/bandwidth and \( y(t) = q(t) \).

Finally, the Drop/Mark Probability Function is of the form

\[
u(t) = p(t) \left[ \frac{1}{\beta_p} \left( 1 + \xi q^{-1} \right) q_{ref}(t+d) - (\xi - \xi_p) q(t) \right]. \quad (9)
\]

The adaptation process takes place in accordance with the formula (3). It should be emphasized that both minimum variance control and inverse problems are not only theoretical. They have been found in many practical aspects [30-33] discussed in detail in [24].

4.2. Closed-loop discrete-time generalized minimum variance adaptive control (DGMVAC)

The DGMVAC law, minimizing the performance index

\[
\min_{u(t)} E \left\{ \left[ \left\| y(t+d) - y_{ref}(t+d) \right\|_p^2 + \left\| u(t) \right\|_Q^2 \right] \right\}, \quad (10)
\]

with \( P(q^{-1}) \) and \( Q(q^{-1}) \) being polynomial matrices, is of the form [24]

\[
u(t) = \left( \bar{Q}(q^{-1}) + \beta_p^2 P(q^{-1}) \bar{F}(q^{-1}) \mathbf{B}(q^{-1}) \right)^{-1} \times \frac{1}{\beta_p} \bar{p}(q^{-1}) \left[ \tilde{\zeta}(q^{-1}) y_{ref}(t+d) - \bar{H}(q^{-1}) y(t) \right], \quad (11)
\]

where \( \bar{F}(q^{-1}) \) and \( \bar{H}(q^{-1}) \) are computed, as for DMVAC, from the polynomial matrix identity (6) and the factorization (7), and \( \beta_p \) is the leading coefficient of \( \mathbf{B}(q^{-1}) \). It is significant that for the \( Q(q^{-1}) = 0 \), \( P(q^{-1}) = I_{n_y} \) our DGMVAC reduces to DMVAC. In order to eliminate the stady-state error \( e_{ss} = y_{ref} - y_{ss} \), where \( y_{ss} \) is the output in the steady-state, the so-called integration action is introduced [24]. Like before, an adaptation process is based on Eqn. (3).

Remark 1. Note that the Drop/Mark Probability Function \( p(t) \) represents the behavior of different subnets/groups of users in terms of lost/retransmitted packets. Due to the use of the nonunique inverses of the nonsquare polynomial matrices we can now, in particular in case of (G)MVC, form the robust \( p(t) \). Therefore, the impact of new method on computer networks bandwidth increase is immediate. Additionally, it is possible in GMVC to obtain the robust \( p(t) \) after using the specially selected control- and control error-weighting polynomial matrices \( Q(q^{-1}) \) and \( P(q^{-1}) \), respectively. Of course, in both cases we preserve the output bandwidth \( y(t+d) = q(t+d) = y_{ref}(t+d) = q_{ref}(t+d) \).

The illustrative DGMVA control system is shown in
5. Simulation examples

The traffic-oriented Discrete Time Simulation Systems (DTSS) can be used to study the behavior of the network. OMNeT++ simulator is an example of such an environment, which is an alternative to the already well-known simulation tools such as NS2, QualNet and Opnet.

It is significant that such an environment should implement adaptive mechanism due to the iterative estimation process of the model parameters. Unfortunately, the aforementioned environments do not offer that option, so the dedicated programming tool Matlab has been chosen. The adaptation process has been simulated with authors’ procedures implementing complex difference equations.

5.1. Discrete-time closed-loop minimum variance adaptive control

The deterministic DMVAC case \( e(t) = 0 \) has been adopted for a model (2) with \( A(q^{-1}) = 1 + 0.8q^{-1}, C(q^{-1}) = 1 + 0.1q^{-1} \) and with a variable parameter \( B(q^{-1}) = B_0 \). It should be noted that the adaptation process is strictly related with the assumed initial simulation parameters. A perfect control for \( \alpha = 0.1 \) and \( \gamma = 1 \) has been achieved for different \( B_0 \) parameters, as shown in Fig. 2. Figure 3 presents \( p(t) \) function.

It is significant that the estimated parameters of model (2) are directly responsible for the network load resulting from its usage. This load is mainly affected by the number of tasks within the available services. Because of the model’s three degrees of freedom, the simulation results are presented only for a changing parameter \( B_0 = B(q^{-1}) \).

The load change during network operation is a classic issue directly related to the non-stationarity of queuing process. Figure 4 shows the deterministic minimum variance adaptive control case, i.e. adaptive perfect control, for a variable load (time – 50 s) and a variable parameter \( B(q^{-1}) = B_0 \) for \( q_{ref}(t) = 100 \). Now we have the steady-state error \( q_{ss} = 0 \) in all cases (for our model with time delay \( d = 1 \)). Of course, a change in the network load corresponds to perturbations of \( p(t) \).

5.2. Discrete-time closed-loop generalized minimum variance adaptive control

Due to the lack of control input constraints in DMVAC, the DGMVA control implementing integration action has been suggested. However, we need...
longer (than DMVAC) time to achieve $q_{ss} = 0$. Application of the DGMVAC algorithm provides the robustification of $p(t)$ in terms of increase of the computer network bandwidth. Figure 5 and 6 show the deterministic DGMVAC case for $Q(q^{-1}) = 0.1$, $P(q^{-1}) = 1$, $\alpha = 0.1$, $\gamma = 1$ and variable $E(q^{-1}) = b_0$. It should be noted that $E(q^{-1})$ and $\zeta(q^{-1})$ are the same as in Example 5.1 and $E(q^{-1}) = 1$, $\bar{H}(q^{-1}) = (c_1 - a_1 + 1) + a_2q^{-1}$ and the polynomial matrix $\bar{A}^p(q^{-1}) = \bar{A}(q^{-1})(1 - q^{-1})$ is substituted for the $\bar{A}(q^{-1})$ one both in Eqn. (2) and the polynomial matrix identity (6).

6. Conclusions

This paper presents a new analytical approach to the problem of efficient packet switching in computer queuing networks. The minimum variance adaptive control and generalized minimum adaptive variance control algorithms have been used for this purpose. The obtained results confirm that solutions presented in this paper can compete with the heuristic methods. The lack of approaches implementing complex analytical solutions for the presented issues creates new possibilities for analysis and synthesis, and above all, for improvement of the networks operation by application of the method described here. It seems natural to transfer achievements presented in the paper in the field of multivariable real systems with a single output and a larger number of inputs, representing different groups of users. Due to the existence of an infinite number of degrees of freedom in such nonsquare MIMO systems, it is possible to design a control system with performance improvement. Moreover, it is certainly possible to use described control algorithms also including those with restricted signal control. There is no doubt, it would have contributed significantly to more efficient operation of computer networks in terms of extending their bandwidth, that means reducing the Drop/Mark Probability Function value. An approach, seen nowhere before, would have been a new contribution to the further research. Currently research is conducted to better represent the reality, while improving the quality of computer networks work. It is significant that the Matlab programming environment successfully fits into the design process of complex control structures and certainly its functionality can support the adaptation process seen there. Due to complexity, the simulation examples concerning the nonsquare MISO systems will be done in the nearest future.

Acknowledgment

Invaluable comments from the anonymous Reviewers are gratefully acknowledged.

References


Received January 2014.

W. P. Hunek, P. Dzierwa