ISSN 1392–124X (print), ISSN 2335–884X (online) INFORMATION TECHNOLOGY AND CONTROL, 2016, T. 45, Nr. 4

Uncertainty and Disturbance Estimator Based Control of Active Suspensions with A Hydraulic Actuator

Lingfei Xiao

College of Energy and Power Engineering, Jiangsu Province Key Laboratory of Aerospace Power Systems, Nanjing University of Aeronautics and Astronautics, Nanjing, China
e-mail: lfxiao@nuaa.edu.cn

Runze Ding

College of Energy and Power Engineering, Jiangsu Province Key Laboratory of Aerospace Power Systems, Nanjing University of Aeronautics and Astronautics, Nanjing, China
crossref *http://dx.doi.org/10.5755/j01.itc.45.4.13372*

Abstract. This paper presents a novel control method for active suspension systems based on the Uncertainty and Disturbance Estimator (UDE) control strategy. The nonlinear dynamics of the hydraulic actuator in a quarter-car active suspension system is considered with uncertainties. In order to facilitate the controller design, the whole system is partitioned into a linear subsystem and a nonlinear subsystem. For the linear subsystem, a reference model is offered based on sky-hook damper and the UDE control strategy is applied to obtain desired fictitious input of linear subsystem. For the nonlinear subsystem, sliding mode control approach is used to construct controller in order to force the output of nonlinear subsystem to track the desired fictitious input of linear subsystem. Simulation results verify the effectiveness of the proposed method.

Keywords: uncertainty and disturbance estimator based control; active suspension systems; hydraulic actuator; sliding mode.

1. Introduction

Suspension systems are important parts of vehicle systems, providing great contribution to the riding comfort and safety of passengers [1]. Usually, there are three types of suspensions: passive, semiactive and active [2–6]. Recently, the control of active suspensions has attracted the attention of many researchers [5–8]. Active suspensions can be used to minimize the vertical forces transmitted to the passengers for riding comfort, and to maximize the tyre-to-road contact for handling and safety [9]. Active suspensions often operate on both smooth roads and rough roads and the perturbation in the parameters is inevitable. For example, the coefficients of damping and stiffness may vary because of ageing.

In active suspensions, actuators can add and dissipate energy from the system so that the attitude of the vehicle can be adjusted through suspension and then the effects of road roughness can be reduced and the riding comfort is improved as a result [10]. Since the actuators pull down or push up together with the suspension motions, the dynamics of actuators should be taken into consideration. For active suspension systems with hydraulic actuators, the sliding mode control (SMC), which is effective for nonlinear and parameter uncertain systems [11, 12], has attracted the interest of many researchers recently [2, 6, 13–17].

A model reference sliding mode controller on the basis of two acceleration sensors for a two degree-of-freedom plant model is investigated in [6]. In [13], a SMC controller was designed for a quarter-car suspension system by taking a sky-hook damper system as a reference model. In these two papers, the actuator dynamics was not taken into consideration. An active suspension system was considered for a quarter-car model using the concept of SMC based on a pneumatic actuator in [2], taking into account the time lag effect of the pneumatic actuator.

In addition to the SMC method, many other control approaches are used for the control of active suspension systems. Rao and Narayanan [18] considered a quarter car vehicle model with sky-hook damper control strategy through linear quadratic regulator (LQR) control scheme. Gao et al. [7] presented a H_∞control for quarter-car active suspension system. Li et al. [19] investigated the non-fragile H∞ controller design problem of a half-vehicle model with active suspension system. However, both of them did not consider the dynamics of the actuator. Ma and Chen [8] developed a disturbance attenuation control of active suspension with nonlinear actuator dynamics, but the effect of uncertainties was not considered.

In this paper, a novel suspension control scheme is derived by means of Uncertainty and Disturbance Estimator (UDE) control approach [21–23] and SMC method, in order to realize the robust control of suspension systems with hydraulic actuator and uncertainties.

The remainder of this paper is organized as follows. Section 2 is the problem formulation, in which the quarter-car model, the dynamics of the hydraulic actuator, the rescaling of states and the repartition of subsystems are given. Section 3 illustrates the design of active suspension controller based on UDE and SMC approaches. Simulation is shown in Section 4. Section 5 draws the conclusions of this paper.

2. Problem Formulation

Consider the quarter-car model of an active suspension system as shown in Fig. 1.

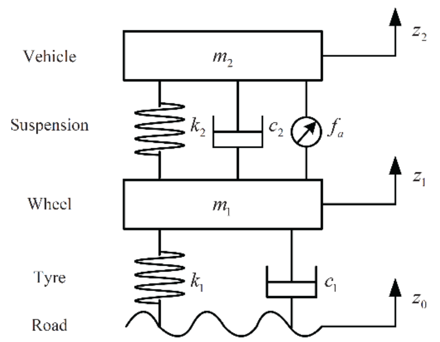

Figure 1. Quarter-car model with an active suspension

In Fig. 1, m_2 is the sprung mass, m_1 is the unsprung mass; k_2 and c_2 are the coefficients of stiffness and damping of the suspension system, respectively; k_1 and c_1 stand for compressibility and damping of the pneumatic tyre, respectively; z_2 and z_1 are the displacements of the sprung and unsprung masses, respectively; z_0 is vertical ground displacements caused by road unevenness; and f_a is the active input force of the suspension system.

The dynamic equations of the sprung and unsprung masses are

$$\begin{aligned} & m_2\ddot{z}_2 + c_2(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_1) = f_a \\ & m_1\ddot{z}_1 + c_2(\dot{z}_1 - \dot{z}_2) + k_2(z_1 - z_2) \\ & \qquad + c_1(\dot{z}_1 - \dot{z}_0) + k_1(z_1 - z_0) = -f_a \end{aligned} . \qquad (1)$$

Since the masses m_i, stiffness coefficients k_i and dumping coefficients c_i $(i = 1, 2)$ of seat suspension system are unavoidable to suffer from perturbation. Assume $m_i = m_{i_0} + \delta m_i$, $k_i = k_{i_0} + \delta k_i$, $c_i = c_{i_0} + \delta c_i$, where $m_{i_0}, k_{i_0}, c_{i_0}$ are the nominal values of m_i, k_i, c_i, respectively, and $\delta m_i, \delta k_i, \delta c_i$ are the uncertainties of m_i, k_i, c_i, respectively, with bounded $\delta m_i, \delta k_i, \delta c_i$.

Usually, the active force f_a is generated by the hydraulic actuator placed between the sprung and unsprung masses. Therefore, a hydraulic actuator of a four-way valve-piston system is regarded in this paper. Detailed introduction of such a hydraulic actuator can be found in [8,9,20] and the following basic concepts are adapted from them.

The active force f_a from the actuator is $f_a = A_r P_L$, where A_r is the piston area of the hydraulic actuator and P_L is the pressure drop across the piston.

The rate of change of P_L can be described as $\dot{P}_L = \alpha Q - \beta P_L - \alpha A_r(\dot{z}_2 - \dot{z}_1)$, where Q is load flow, $\alpha = \frac{4\beta_e}{V_t}$, V_t is total actuator volume and β_e is effective bulk modulus, $\beta = \alpha C_{tp}$, C_{tp} is the total piston leakage coefficient of the piston.

The servovalve load flow equation is given by

$$Q = \frac{C_d v}{\sqrt{\rho}} x_v \omega \qquad (2)$$

where

$$\omega = \mathrm{sgn}[P_s - \mathrm{sgn}(x_v)P_L]\sqrt{|P_s - \mathrm{sgn}(x_v)P_L|}$$

$P_s > 0$ is supply pressure, x_v is valve displacement from its 'closed' position, C_d is discharge coefficient, v is spool valve area gradient, ρ is hydraulic fluid density.

The spool valve displacement is controlled by the input to the servovalve μ, which could be a current or a voltage. The valve dynamic is approximated by a linear filter with time constant τ, namely, $\dot{x}_v = \frac{1}{\tau}(-x_v + u)$.

It is worth pointing out that the scale of P_L is about 10^7 while the scale of Q is around 10^{-3}, their scales differ greatly. Thus, rescaling P_L and Q is very important to reduce the numerical integration errors in simulations. Besides, in view of actuator saturation, it is reasonable to assume the load flow Q is bounded by Q_s. Hence, $\bar{Q} = \frac{Q}{Q_s}$ and $\bar{P}_L = \frac{P_L}{P_s}$ are introduced. Then

$$\begin{aligned}\dot{\bar{P}}_L &= \frac{\alpha}{P_s}Q_s\bar{Q} - \beta\bar{P}_L - \frac{\alpha}{P_s}A_r(\dot{z}_2 - \dot{z}_1)\\ \bar{Q} &= \frac{C_d v}{\sqrt{\rho}}\frac{\sqrt{P_s}}{Q_s}x_v\bar{\varpi}\end{aligned} \tag{3}$$

where $\bar{\varpi} = \mathrm{sgn}[1-\mathrm{sgn}(x_v)\bar{P}_L]\sqrt{|1-\mathrm{sgn}(x_v)\bar{P}_L|}$.

According to the second sub-equation in (3) and valve dynamic, the derivative of $\bar{Q}$ is

$$\dot{\bar{Q}} = \frac{C_d v}{\sqrt{\rho}}\frac{\sqrt{P_s}}{Q_s}\Big[\frac{1}{\tau}(-x_v + u)\bar{\varpi} - \frac{1}{2}\Big|\frac{x_v}{\bar{\varpi}}\Big|\dot{\bar{P}}_L\Big]. \tag{4}$$

Obviously, the dynamic of the quarter-car active suspension with actuator dynamic system can be described by (1) and (3). So, an intuitional way to design controller is to divide the system into two subsystems which are based on (1) and (3). However, the first sub-equation in (3) is linear with respect to $\bar{Q}$, $\bar{P}_L$ and $\dot{z}_2$, $\dot{z}_1$, while (4) is with strong nonlinearity. The controller design directly in accordance with (3) will be a tough task.

Therefore, in this paper, the whole system is divided into another two subsystems, one is linear subsystem and the other is nonlinear subsystem. In the nonlinear subsystem, there is only one state dynamic equation. Hence, the controller design complexity can be reduced greatly.

Select $\boldsymbol{x} = [z_2, z_1, \dot{z}_2, \dot{z}_1, \bar{P}_L]^{\mathrm{T}}$, $\xi = \bar{Q}$, $\boldsymbol{Z}_0 = [z_0, \dot{z}_0]$ the active suspension system with hydraulic actuator dynamics is

$$\begin{aligned}\dot{\boldsymbol{x}} &= A\boldsymbol{x} + B\xi + B_z\boldsymbol{Z}_0\\ &\quad+(\Delta A\boldsymbol{x} + \Delta B\xi + \Delta B_z\boldsymbol{Z}_0)\\ \dot{\xi} &= f(\boldsymbol{x},\xi) + g(\boldsymbol{x},\xi)u\\ &\quad+\big(\Delta f(\boldsymbol{x},\xi) + \Delta g(\boldsymbol{x},\xi)u\big)\end{aligned} \tag{5}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ -\frac{k_{2_0}}{m_{2_0}} & \frac{k_{2_0}}{m_{2_0}} & -\frac{c_{2_0}}{m_{2_0}} & \frac{c_{2_0}}{m_{2_0}} & \frac{A_r P_s}{m_{2_0}}\\ \frac{k_{2_0}}{m_{1_0}} & \frac{-k_{2_0}-k_{1_0}}{m_{1_0}} & \frac{c_{2_0}}{m_{1_0}} & \frac{-c_{2_0}-c_{1_0}}{m_{1_0}} & -\frac{A_r P_s}{m_{1_0}}\\ 0 & 0 & -\frac{\alpha A_r}{P_s} & \frac{\alpha A_r}{P_s} & -\beta \end{bmatrix},$$

$$B = \Big[0,0,0,0,\frac{\alpha Q_s}{P_s}\Big]^{\mathrm{T}}, B_z = \begin{bmatrix} 0,0,0,\frac{k_{1_0}}{m_{1_0}},0\\ 0,0,0,\frac{c_{1_0}}{m_{1_0}},0\end{bmatrix}^{\mathrm{T}},$$

$$f = \frac{\lambda}{\alpha}\frac{\sqrt{P_s}}{Q_s}\Big[-\frac{1}{\tau}x_v\bar{\varpi} - \frac{1}{2}\Big|\frac{x_v}{\bar{\varpi}}\Big|\dot{\bar{P}}_L\Big],$$

$$g = \frac{\lambda}{\alpha}\frac{\sqrt{P_s}}{Q_s}\frac{1}{\tau}\bar{\varpi}$$

where $\Delta A, \Delta B, \Delta B_w, \Delta C, \Delta f, \Delta g$ are the uncertainties of A, B, B_w, C, f, g, respectively. $\lambda = \alpha\frac{C_d v}{\sqrt{\rho}}$ and x_v is taken as a function of ξ according to servo valve load flow equation.

Usually, there are two kinds of road excitations. One is the case that an isolated bump in an otherwise smooth road surface. The corresponding ground displacement can be given by

$$z_0(t) = \begin{cases} \frac{A_{z0}}{2}\big(1 - cos(\frac{2\pi v}{L_{z0}}t)\big), & if \quad 0 \le t \le \frac{A_{z0}}{L_{z0}}\\ 0, & if \quad t \ge \frac{A_{z0}}{L_{z0}}\end{cases}$$

where A_{z0} and L_{z0} are the height and the length of the bump, $\boldsymbol{v}$ is the vehicle forward velocity. The other is to consider the road excitation z_0 as a vibration, which is consistent and typically specified as random process with a ground displacement power spectral density (PSD) of $G_q(f) = 4\pi^2 G_q(n_0)n_0^2 v$, where $G_q(n_0)$ stands for the road roughness coefficient, n_0 is the reference spatial frequency, and v is the vehicle forward velocity [8]. It is reasonable to assume that $\boldsymbol{Z}_0$ is available.

3. Active suspension controller design

Obviously, (5) has two subsystems, one is the linear subsystem which is described by the first sub-equation in (5), and the other is the nonlinear subsystem which is described by the second sub-equation in (5). Therefore, the procedure of obtaining controller includes two steps.

Firstly, ξ can be seen as a fictitious input for the linear subsystem, namely, the first sub-equation in (5), an auxiliary control ξ_d is designed to be the desired signal for ξ on the basis of Uncertainty and Disturbance Estimator (UDE) control approach ([21-23]).

Secondly, because sliding mode control (SMC) method a kind of strong robust strategies which is convenient to be used for nonlinear system, according to the nonlinear subsystem, namely, the second sub-equation in (5), control input u is determined based on SMC such that the actual ξ tracks the desired ξ_d well.

For linear subsystem in (5), let $\mathbf{d} = \Delta A x + \Delta B\xi + \Delta B_z\boldsymbol{Z}_0$ represent lumped uncertainty. Then

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\xi + B_z\mathbf{Z}_0 + \mathbf{d}. \tag{6}$$

The reference model of suspension system should have satisfying dynamic.

The following reference model (7)

$$\dot{\boldsymbol{x}}_m = A_m \boldsymbol{x}_m + B_z \mathbf{Z}_0 \tag{7}$$

is used as the reference model, which is on the basis of sky-hook damper [24] and as shown in Fig. 2.

In (7) $\mathbf{x}_m = [z_{2_m}, z_{1_m}, \dot{z}_{2_m}, \dot{z}_{1_m}, \bar{P}_L]^{\mathrm{T}}$ and

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\dfrac{k_{2_0}}{m_{2_0}} & \dfrac{k_{2_0}}{m_{2_0}} & \dfrac{c_{sh} - c_{2_0}}{m_{2_0}} & \dfrac{c_{2_0}}{m_{2_0}} & \dfrac{A_r P_s}{m_{2_0}} \\ \dfrac{k_{2_0}}{m_{1_0}} & \dfrac{-k_{2_0} - k_{1_0}}{m_{1_0}} & \dfrac{c_{2_0} - c_{sh}}{m_{1_0}} & \dfrac{-c_{2_0} - c_{1_0}}{m_{1_0}} & -\dfrac{A_r P_s}{m_{1_0}} \\ 0 & 0 & -\dfrac{\alpha A_r}{P_s} & \dfrac{\alpha A_r}{P_s} & -\beta \end{bmatrix},$$

with

$$c_{sh} = \begin{cases} c_{sky}, & if \quad \dot{z}_{1_m}(\dot{z}_{2_m} - \dot{z}_{1_m}) \geq 0 \\ 0, & if \quad \dot{z}_{1_m}(\dot{z}_{2_m} - \dot{z}_{1_m}) < 0 \end{cases}$$

and $c_{sky} > 0$ is the coefficients of sky-hook damper.

Let $C_{sh} = A_m - A$,

then
$$C_{sh} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{c_{sh}}{m_2} & 0 & 0 \\ 0 & 0 & -\dfrac{c_{sh}}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The control objective is to force the error $\boldsymbol{e}_x = \boldsymbol{x}_m - \boldsymbol{x}$ to be stable and with specified dynamic, that is

$$\dot{\boldsymbol{e}}_x = \dot{\boldsymbol{x}}_m - \dot{\boldsymbol{x}} = (A_m + K)\boldsymbol{e}_x \tag{8}$$

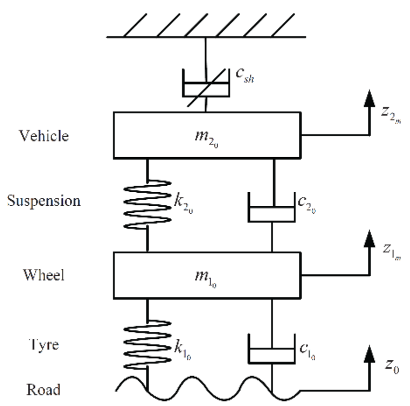

Figure 2. Sky-hook damper reference model

where K is an error feedback gain matrix with appropriate dimensions.

The error dynamics can be changed via adjusting the error feedback gain K. A bigger absolute value of K results in faster error dynamics [23]. The selection of K can be based on classical control theory method, such as pole placement, in order to guarantee stability and desired dynamic.

According to (6), (7) and(8), there exists

$$\begin{aligned} & A_m \mathbf{x}_m + B_z \mathbf{Z}_0 - (A\mathbf{x} + B\xi + B_z \mathbf{Z}_0 + \mathbf{d}) \\ & = (A_m + K)\mathbf{e}_x \end{aligned} .4$$

Thus, the auxiliary control ξ_d can be yielded as

$$\begin{aligned} B\xi_d &= A_m \boldsymbol{x}_m - A\boldsymbol{x} - \mathbf{d} - (A_m + K)\boldsymbol{e}_x \\ &= A_m(\boldsymbol{e}_x + \boldsymbol{x}) - A\boldsymbol{x} - \mathbf{d} - (A_m + K)\boldsymbol{e}_x . \\ &= (A_m - A)\boldsymbol{x} - \mathbf{d} - K\boldsymbol{e}_x \\ &= C_{sh}\boldsymbol{x} - K\boldsymbol{e}_x - \mathbf{d} \end{aligned} \tag{9}$$

Because there exists the unknown term $-\mathbf{d}$ in(9), the auxiliary control signal ξ_d cannot be obtained directly from (9). The UDE based control strategy proposed in [21] adopts an estimation of this unknown term to construct control law.

Let $\mathbf{u}_d = -\mathbf{d}$, assume that $g_f(t)$ is the impulse response of a strictly proper filter $G_f(s)$, whose passband contains the frequency content of $\mathbf{u}_d$. Then $\mathbf{u}_d$ can be accurately estimated from the output of the UDE as

$$\mathbf{ude} = \mathbf{u}_d \star g_f(t) \tag{10}$$

where ‘ $\star$ ’ is the convolution operator, with $G_f(s) = \mathcal{L}\{g_f(t)\}$ and $\mathcal{L}\{\cdot\}$ is the Laplace transform operator.

Due to linear subsystem dynamics (6), $\mathbf{u}_d$ can be represented as

$$\mathbf{u}_d = A\mathbf{x} + B\xi + B_z \mathbf{Z}_0 - \dot{\boldsymbol{x}} . \tag{11}$$

Thus, according to (9) and (11), the auxiliary control action can be yielded by

$$\begin{aligned} & B\xi_d = C_{sh}\boldsymbol{x} - K\boldsymbol{e}_x + (A\boldsymbol{x} + B\xi + B_z \mathbf{Z}_0 - \dot{\boldsymbol{x}}) \star g_f(t) \\ & \Rightarrow \xi_d = B^{+}\left[-A\boldsymbol{x} - \mathcal{L}^{-1}\left\{ sG_f(s)\left(1 - G_f(s)\right)^{-1} \right\}\boldsymbol{x} \right. \\ & \quad + \mathcal{L}^{-1}\left\{ G_f(s)\left(1 - G_f(s)\right)^{-1} \right\} B_z \mathbf{Z}_0 \\ & \quad \left. + \mathcal{L}^{-1}\left\{ \left(1 - G_f(s)\right)^{-1} \right\} (A_m \boldsymbol{x} - K\boldsymbol{e}_x) \right] \end{aligned} \tag{12}$$

where $B^{+} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}$ is the pseudo-inverse of B.

Obviously, the auxiliary control ξ_d has nothing to do with the unknown terms.

Now, consider the nonlinear subsystem in (5). Let the error between the actual ξ and the desired ξ_d be

$$e_{\xi} = \xi - \xi_d . \tag{13}$$

According to sliding mode control (SMC) theory, a sliding surface with desired performance should be created first of all, and then a suitable control law is required to drive states to origin along with the sliding surface.

Therefore, consider sliding mode variable

$$s_{cn} = e_{\xi}. \tag{14}$$

Due to reaching law approach, the control input is constructed as

$$u = g^{-1}\left[-q_1 s_{cn} - q_2 \operatorname{sgn}(s_{cn}) - \dot{\xi}_d - f\right]$$

where $q_1 > 0, q_2 > 0$ are designable parameters.

In order to reduce chattering, $\frac{2}{\pi}\arctan(\frac{s_{cn}}{\delta})$ is used to replace $\operatorname{sgn}(s_{cn})$, then the control input is

$$u = g^{-1}\left[-q_1 s_{cn} - q_2 \frac{2}{\pi}\arctan\left(\frac{s_{cn}}{\delta}\right) - \dot{\xi}_d - f\right]. \tag{15}$$

4. Simulation

In this section, the simulation results will be given on a quarter-car model [25], in which parameters are $m_{2_0} = 320\ kg$, $m_{1_0} = 40 kg$,

$k_{2_0} = 18\ kN/m$, $k_{1_0} = 200\ kN/m$ ·

$c_{2_0} = 1\ kN \cdot s/m, c_{1_0} = 10\ kN \cdot s/m$

Physical parameters for the hydraulic actuator are given as [9]: $\beta = 1(1/s)$, $Ar = 3.35 \times 10.4 m^2$, $\tau = 1/30 s$, $P_s = 1.03425 \times 10^7 Pa$, $Q_s = 1.5563 \times 10^{-3} m^3/s$, $\alpha = 4.515 \times 10^{13} N/m^5$, $\lambda = \alpha C_d v \sqrt{1/\rho} = 1.545 \times 10^9 N/(m^{5/2} kg^{1/2})$.

Suppose $y = [z_1, \ddot{z}_2]^{\mathrm{T}}$ are measurable outputs, and consider the parameters of suspension system suffer from perturbations $\delta k_2 = 50\% k_{2_0}$, $\delta c_2 = -50\% c_{2_0}$, $\delta c_1 = -50\% c_{1_0}$.

Select $c_{sky2_0} = 2\ kN \cdot s/m$.

Assume that the frequency range of the suspension system dynamic and the external disturbances is limited by ω_f. Although the low-pass filter $G_f(s)$ can be chosen by designers arbitrarily, however, it is practical to select $G_f(s)$ to be of a simple form such as $G_f(s) = \frac{1}{Ts+1}$, where $T = \frac{1}{\omega_f} > 0$. Therefore, in this simulation $T = 0.01$ is selected.

In order to evaluate the performance of the designed closed-loop active suspension system, we consider two typical cases as introduced in Section 2.

Case 1: Consider the case of an isolated bump in an otherwise smooth road surface. Assume $A_{z0} = 0.1m$, $L_{z0} = 5m$, and $v = 30 km/h$. The corresponding road excitation is shown in Fig. 3.

Case 2: Consider road excitation z_0 as a vibration, select the road roughness as $G_q(n_0) = 256 \times 10^{-6} m^3$, $n_0 = 0.1$, which corresponds to very poor ground, assume $v = 30 km/h$.

Let the designable parameters are $K = \operatorname{diag}\{-10^4, -10^4, -10^4, -10^4, -200\}$, $q_1 = 0.01$, $q_2 = 0.01$, $\delta = 2.5 \times 10^{-6}$.

Simulation results are shown in Fig. 3-Fig. 8.

According to Fig. 3-Fig. 6, it is obvious that under the proposed method, the suspension acceleration $\ddot{z}_2$, the input to the servo valve u, the input to the linear subsystem ξ and the tracking error e_{ξ} converge quickly and have good precision. Although the tyre displacement z_1 and the tracking error e_{x_1} have ups and downs at the end of simulation, they converge quickly and the volatility is relatively small when compared with the amplitude of road excitation.

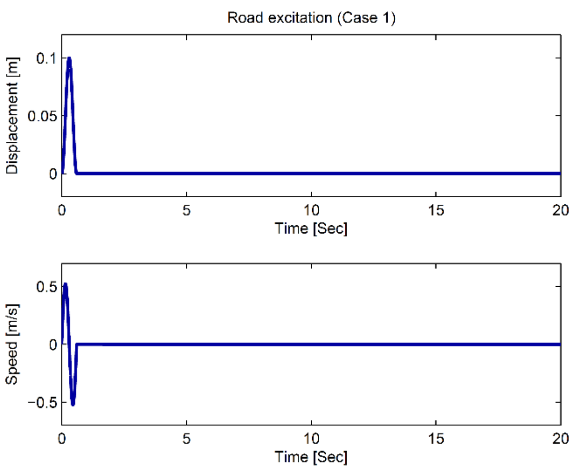

Figure 3. Road excitation (Case 1)

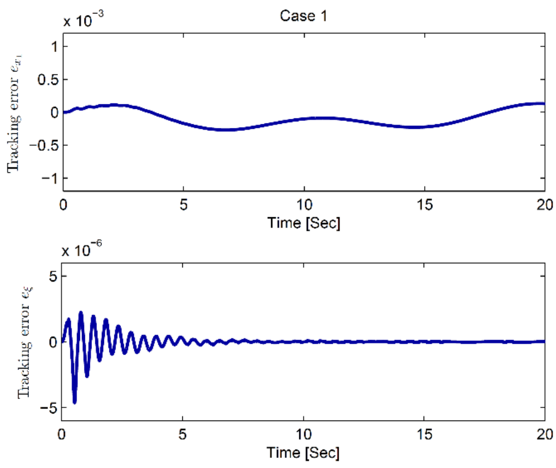

Figure 4. Tracking error (e_{x_1}, e_{ξ}) (Case 1)

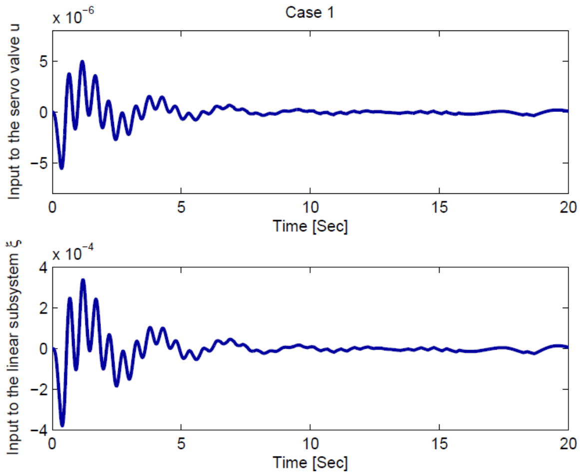

Figure 5. Input to the servo valve u and input to the linear subsystem ξ (Case 1)

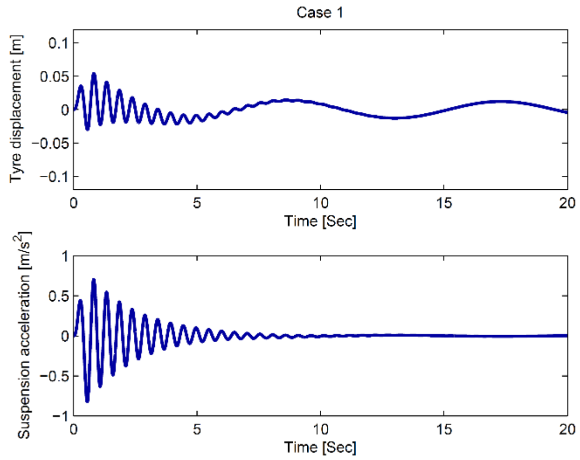

Figure 6. Tyre displacement z_1 and suspension acceleration $\ddot{z}_2$ (Case 1)

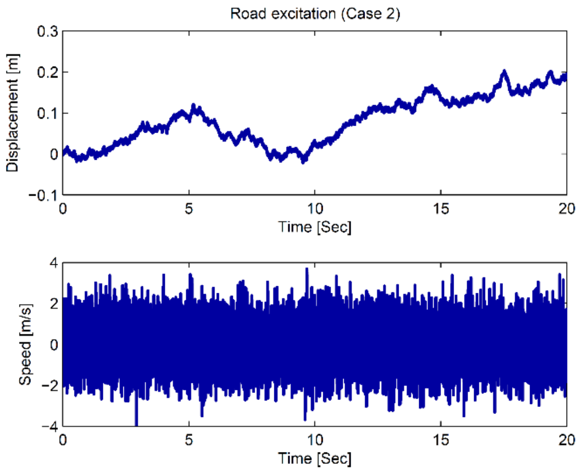

Figure 7. Road Excitation(Case2)

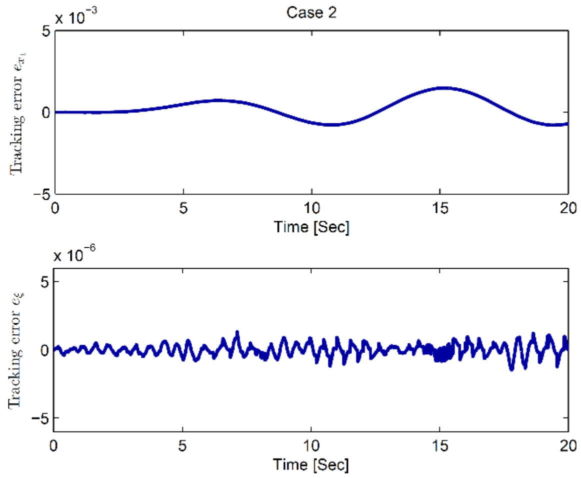

Figure 8. Tracking error (e_{x_1}, e_ξ) (Case 2)

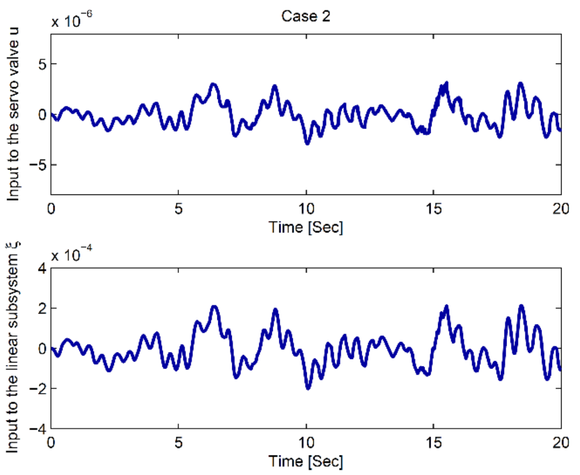

Figure 9. Input to the servo valve u and input to the linear subsystem ξ (Case 2)

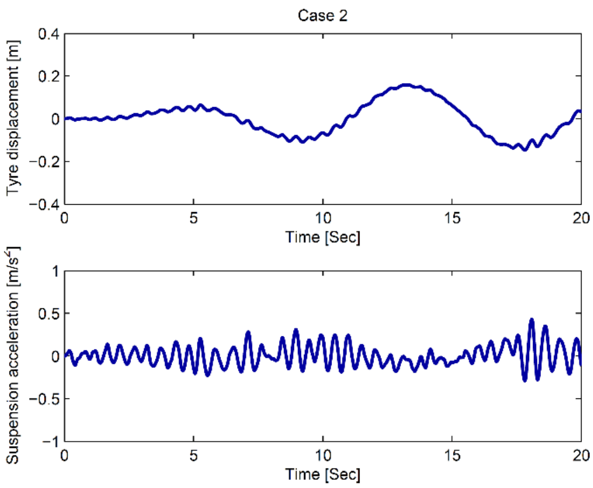

Figure 10. Tyre displacement z_1 and suspension acceleration $\ddot{z}_2$ (Case 2)

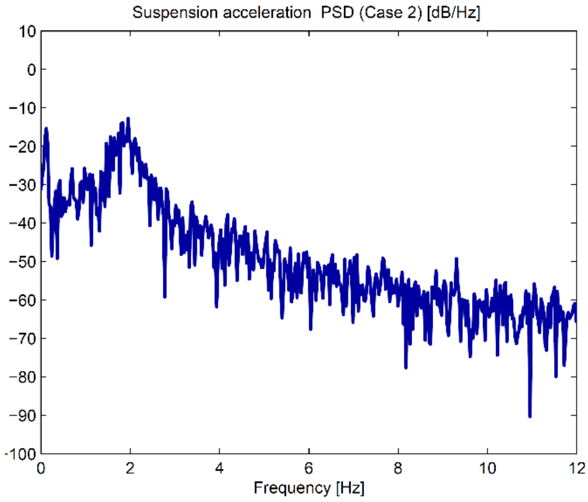

Figure 11. Suspension acceleration PSD (Case 2)

From Fig. 7-Fig. 11, one can find that the curves of the tyre displacement z_1, the suspension acceleration $\ddot{z}_2$, the input to the servo valve u, the input to the linear subsystem ξ, the tracking errors e_{x_1} and e_ξ are much more smoother than the road excitation. The power spectral density (PSD) of $\ddot{z}_2$ has low frequency band 4-8 Hz, which is the widely accepted ride comfort frequency range.

Therefore, the presented method possesses good performance in the whole.

5. Conclusions

For a quarter-car active suspension system with uncertainties, a novel robust control method is presented. For the nonlinear dynamics of hydraulic actuator, the whole system is repartitioned into a linear subsystem and a nonlinear subsystem, instead of dividing into actuator subsystem and suspension subsystem. The repartition facilitates the controller design greatly. For the linear subsystem, a reference model is offered based on sky-hook damper at first, and then the Uncertainty and Disturbance Estimator (UDE) control approach is used to get desired fictitious input of linear subsystem. For the nonlinear subsystem, sliding mode control (SMC) strategy is employed to construct controller in order to force the output of nonlinear subsystem to track the desired fictitious input of linear subsystem. Simulation on two kinds of road surfaces are given, the results verify that the proposed method has good performance.

Acknowledgments

This work is supported by the Fundamental Research Funds for the Central Universities, NO.NS2016027.

The authors would like to thank Professor Qing-Chang Zhong, all of team-mates and anonymous reviewers for their helpful suggestions.

References

[1] **S. B. Choi, Y. M. Han**. Vibration control of electrorheological seat suspension with human-body model using sliding mode control. *Journal of Sound and Vibration*, 2007, Vol. 303, No. 1-2, 391–404.

[2] **T. Yoshimura, A. Kume, M. Kurimoto, J. Hino.** Construction of an active suspension system of a quarter car model using the concept of sliding mode control. *Journal of Sound and Vibration*, 2001, Vol. 239, No. 2, 187–199.

[3] **Y. Wu, H. Su, R. Lu, Z.-G. Wu, Z. Shu**. Passivity-based non-fragile control for Markovian jump systems with a periodic sampling. *Systems & Control Letters*, 2015, Vol. 84, 35–43.

[4] **G Verros, S Natsiavas, C. Papadimitriou.** Design optimization of quarter car models with passive and semi-active suspensions under random road excitation. *Journal of Vibration and Control*, 2005, Vol. 11, No. 5, 581–606.

[5] **N Yagiz, Y. Hacioglu.** Backstepping control of a vehicle with active suspensions. *Control Engineering Practice*, 2008, Vol. 16, 1457–1467.

[6] **J. L. Yao, W. K. Shi, J. Q. Zheng, H. P. Zhou, J. L. Yao**. Development of a sliding mode controller for semi-active vehicle suspensions. *Journal of Vibration and Control*, 2013, Vol. 19, No. 8, 1152–1160.

[7] **H. Gao, W. Sun, P. Shi**. Robust sampled-data H∞ control for vehicle active suspension systems. *IEEE Transactions on Control Systems Technology*, 2010, Vol. 18, No. 1, 238–245.

[8] **M. M. Ma, H. Chen.** Disturbance attenuation control of active suspension with non-linear actuator dynamics. *IET Control Theory and Applications*, 2011, Vol. 5, No. 1, 112–122.

[9] **J. S. Lin, I. Kanellakopoulos**. Nonlinear design of active suspensions. *IEEE Control Systems*, 1997, Vol. 17, No. 3, 45–59.

[10] **W. Sun, H. Gao, B. Yao**. Adaptive robust vibration control of full-car active suspensions with electrohydraulic actuators. *IEEE Transactions on control systems technology*, 2013, Vol. 21, No. 6, 2417–2422.

[11] **J. Y. Hung, W. B. Gao, J. C. Hung**. Variable structure control: a survey. *IEEE Transactions on Industrial Electronics*, 1993, Vol. 40, No. 3, 2–22.

[12] **W. B. Gao.** Theoretical Foundation for Variable Structure Control. *Beijing, China Science and Technology Press*, 1998.

[13] **C. Kim, P. I. Ro**. A sliding mode controller for vehicle active suspension systems with non-linearities. In: *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 1998, Vol. 212, pp. 79–92.

[14] **S. Benelghali, H. B. M. El, J. F. Charpentier, T. Ahmed-Ali, I. Munteanu**. Experimental validation of a marine current turbine simulator application to a permanent magnet synchronous generator-based system second-order sliding mode control. *IEEE Transactions on Industrial Electronics*, 2011, Vol. 58, No. 1, 118–126.

[15] **Z. G. Wu, P. Shi, H. Su, J. Chu.** Asynchronous I2−I∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities. *Automatic*a, 2014, Vol. 50, No. 1, 180–186.

[16] **H. Lin, H. Su, Z. Shu, Z.-G. Wu, Y. Xu**. Optimal estimation in UDP-like networked control systems with intermittent inputs: stability analysis and suboptimal filter design. *IEEE Transactions on Automatic Control*, 2015, Vol. 61, No. 7, 1794 - 1809.

[17] **R. J. Lian**. Enhanced adaptive self-organizing fuzzy sliding-mode controller for active suspension systems. *IEEE Transactions on Industrial Electronics*, 2013, Vol. 60, No. 3, 959–969.

[18] **L. V. V. Gopala Rao, S. Narayanan**. Sky-hook control of nonlinear quarter car model traversing rough road matching performance of LQR control. *Journal of Sound and Vibration*, 2009, Vol. 323, No. 3-5, 515–529.

[19] **H. Li, H. Liu, C. Hilton, S. Hand.** Non-fragile H_1 control for half-vehicle active suspension systems with actuator uncertainties. *Journal of Vibration and Control*, 2012, Vol. 19, No. 4, 560–575.

[20] **I. Fialho, G.J. Balas**. Road adaptive active suspension design using linear parameter-varying gain-scheduling. *IEEE Transactions on Control Systems Technology*, 2002, Vol. 10, No. 1, 43–54.

[21] **Q. Zhong, D. Rees.** Control of uncertain LTI systems based on an uncertainty and disturbance estimator. *Journal of Dynamic Systems Measurement & Contro*l, 2004, Vol. 126, No. 4, 905-910.

[22] **A Kuperman, Q. Zhong**. Control of uncertain linear systems with a state delay based on an uncertainty and disturbance estimator. In: *Proceedings of Sixth IFAC Symposium on Robust Control Design, Haifa, Israel,* 2009, pp. 279–283.

[23] **A Kuperman, Q. Zhong.** Robust control of uncertain nonlinear systems with state delays based on an uncertainty and disturbance estimator. *International Journal of Robust and Nonlinear Control,* 2011, Vol. 21, No. 1, 79-92.

[24] **M. Nagai.** Recent researches on active suspensions for ground vehicles. *The Japan society mechanical engineers, Series C,* 1993, Vol. 36, No. 2, 161–170.

[25] **W. Sun, Y. Zhao, J. Li, L. Zhang, H. Gao.** Active suspension control with frequency band constraints and actuator input delay. *IEEE Transactions on Industrial Electronics,* 2012, Vol. 59, 530–537.

Received January 2016.